U	is the dimensional velocity of the "quasisolid" core;
τ_0	is the yield stress;
V	is the dimensional velocity at the inlet;
Q	is the flow rate;
ξ, η	are the dimensionless coordinates;
w, φ	are the dimensionless velocity components;
s	is the plasticity parameter;
Re	is the Reynolds number;
W	is the dimensionless velocity of the "quasisolid" core.

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THERMAL INTERACTION BETWEEN GAS LINE AND FROZEN SOILS

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UDC 536.244

The article examines the numerical solution of the problem of heat exchange during the flow of gas through an underground pipeline taking into account the phase transitions in the soil under various cooling regimes of the gas.

Investigations of the thermal regimes of pipelines running through frozen soil are dealt with in many works [1-6]. These works give most attention to the investigation of the thermal fields of the soil, while the temperature of the pumped medium is taken as constant.

The present work examines the two-dimensional problem of the change of gas along the pipeline and in time, taking into account the dynamics of heat exchange with the environment and of phase transitions in the soil.

The examined problem includes two groups of equations. The first expresses the laws of conservation for a gas moving in the gas pipe, and with the usually adopted assumptions [7], it can change to the form

$$\frac{\beta_0}{w} T_t + T_x = \beta T + \beta_1 \alpha \vartheta |_{r_0} + \beta_3, \quad \frac{1}{w} M_t + M_x = -(\ln w)_x, \quad P_x = -(z_0 R_0)^{-1} \left(\frac{\lambda}{4r_0} w^2 + g \sin \mu \right) \exp M. \tag{1}$$

We adopt the following boundary conditions:

$$\begin{gathered} T|_{t=0} = T_1(x), \quad T|_{x=0} = T_2(t); \quad P|_{t=0} = P_1(x), \\ P|_{x=0} = P_2(t), \quad 0 \leqslant x \leqslant L, \quad 0 \leqslant t \leqslant t_f. \end{gathered} \tag{2}$$

The equations of the second group describe the distribution of the temperature field of the soil around the pipeline [9, 11]

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$$H(\vartheta)_t = (K\vartheta_x)_x + \frac{1}{r}(rK\vartheta_r)_r + \frac{1}{r^2}(Kv_\varphi)_\varphi, \tag{3}$$

$$(0 < x < L, \quad 0 < t \leqslant t_f, \quad r_0 < r < r_2, \quad \varphi' \leqslant \varphi \leqslant \varphi'',$$

$$r_0 < r < r_1, \quad 0 \leqslant \varphi \leqslant \varphi', \quad \varphi'' \leqslant \varphi \leqslant 2\pi),$$

$$\vartheta|_{t=0} = \vartheta_0(x, r, \varphi),$$

$$(0 \leqslant x \leqslant L, \quad r_0 \leqslant r \leqslant r_1, \quad 0 \leqslant \varphi \leqslant \varphi',$$

$$\varphi'' < \varphi \leqslant 2\pi, \quad r_0 \leqslant r \leqslant r_2, \quad \varphi' \leqslant \varphi \leqslant \varphi''),$$

$$K\vartheta_x|_{x=0} = \psi_1, \quad K\vartheta_x|_{x=L} = \psi_2,$$

$$\vartheta|_{r=R(x,t)} = \vartheta^*, \quad \vartheta|_{\substack{\varphi=\varphi'-0 \\ \varphi=\varphi''-0}} = \vartheta|_{\substack{\varphi=\varphi'+0 \\ \varphi=\varphi''+0}},$$

$$K\vartheta_r|_{r=r_0} = \alpha(\vartheta|_{r_0} - T),$$

$$K\frac{\partial\vartheta}{\partial n}\bigg|_{r=r_1} = \alpha_a(T_a - \vartheta|_{r_1}), \quad 0 \leqslant \varphi \leqslant \varphi', \quad \varphi'' \leqslant \varphi \leqslant 2\pi,$$

$$K\vartheta_r|_{r=r_2} = \vartheta_1, \quad \varphi' \leqslant \varphi \leqslant \varphi'',$$

$$K\vartheta_r|_{r_0+\delta_{me}-0} = K\vartheta_r|_{r_0+\delta_{me}+0}, \quad \vartheta|_{r_0+\delta_{me}-0} = \vartheta|_{r_0+\delta_{me}+0},$$

where

$$K = \begin{cases} K_{me} & r_0 \leqslant r \leqslant r_0 + \delta_{me}, \\ K_2, & r_0 + \delta_{me} < r < R, \\ K_1, & R < r < r_i, \end{cases}$$

$$H(\vartheta) = \int_0^\vartheta [B(\xi) + \varkappa\delta(\xi - \vartheta^*)]\, d\xi,$$

$$B(\vartheta) = \begin{cases} c_{me}\gamma_{me} & r_0 \leqslant r \leqslant r_0 + \delta_{me}, \\ c_2\gamma, & r_0 + \delta_{me} < r < R, \\ c_1\gamma, & R < r < r_i, \quad i = \begin{cases} 1, & 0 \leqslant \varphi \leqslant \varphi', \\ 2, & \varphi'' < \varphi < 2\pi, \\ & \varphi' \leqslant \varphi \leqslant \varphi'', \end{cases} \end{cases}$$

$$\varphi' = \arccos\left(\frac{r_0 + r_1}{r_2 + r_0}\right), \quad \varphi'' = \varphi'.$$

It was shown [9-11] that the solution of the first equation of system (1) under condition (2) and with $\tilde{w} = w(x) = w(x, t)$, $\tilde{t} \in [0, t]$ has the form

$$\tilde{T}(x, t) = \begin{cases} T_1\left(x - \frac{\tilde{w}t}{\beta_0}\right)\exp\left(\int_{x-\frac{\tilde{w}t}{\beta_0}}^{x} \tilde{\beta}\, d\xi\right), & x \geqslant \tilde{w}t/\beta_0, \\ T_2\left(t - \frac{x\beta_0}{\tilde{w}}\right)\exp\left(\int_0^x \tilde{\beta}\, d\xi\right) + \int_0^x \left[\tilde{\beta}_3 + \tilde{\beta}_1\alpha\vartheta\left(t - \frac{x - \xi}{\tilde{w}}\beta_0\right)\right] \\ \quad \times \exp\left(\int_\xi^x \tilde{\beta}\, d\xi\right) d\xi, & x < \frac{\tilde{w}t}{\beta_0}. \end{cases} \tag{4}$$

Using the method of [11], we present the solution of the second equation of (1) under condition (2) and with $\tilde{w} = \tilde{w}(x) \equiv w(x, \tilde{t})$, $\tilde{t} \in [0, t]$ in the form

TABLE 1. Dynamics of the Changes in Air Temperature and Height of Snow Cover

Month	1	2	3	4	5	6	7	8	9	10	11	12
T_a, °C	−23,6	−22,2	−18,3	−9,4	−1,6	7,8	1,38	1,6	5,4	−3,7	−15,3	−21,3
h_{sn}, m	0,3	0,4	0,4	0,45	0,2	—	—	—	0,05	0,1	0,2	0,25

$$M=\begin{cases}\ln\dfrac{P_1(x-\tilde{w}t)}{T_1(x-\tilde{w}t)}, & x\geqslant\tilde{w}t,\\[2ex] \ln\dfrac{P_2\left(t-\int\limits_0^x\tilde{w}^{-1}\,d\xi\right)}{T_2\left(t-\int\limits_0^x\tilde{w}^{-1}\,d\xi\right)}+\int\limits_0^x\dfrac{\ln\tilde{w}}{\tilde{w}}\,d\xi-\ln\tilde{w}, & x<\tilde{w}t.\end{cases}\tag{5}$$

Then the pressure is determined by the expression

$$P(x,\ t)=\exp MT(x,\ t).\tag{6}$$

From the third equation of (1) we find

$$w(x,\ t)=\left[\ |z_0R_0\exp(-M)P_x+g\sin\mu|\ \frac{4r_0}{\pi}\right]^{0.5}.\tag{7}$$

Since w is contained in (4)-(6), iteration is necessary to obtain T, M, and P. With zero iteration, we take the value of w on the previous time-dependent layer. Using the method of [11], we can show that

$$\|T^s-T^{s-1}\|_{L_2}\leqslant\text{const}\,\|w^s-w^{s-1}\|_{L_2},\quad s=0,\ 1,\ 2,\ \ldots\tag{8}$$

For underground gas lines of small diameter, the effect of heat exchange on the soil surface is negligibly small [1], and we can therefore write

$$(K\vartheta_\varphi)_\varphi\cong 0,\quad K\frac{\partial\vartheta}{\partial n}\Big|_{r_1}\cong 0,\quad 0\leqslant\varphi\leqslant 2\pi,$$

and it is expedient to solve the problem (3) in the region $0\le x\le L$, $r_0\le r\le r_1$, $0\le t\le t_f$.

For pipelines with large diameter placed at small depth, the influence of the soil surface must be taken into account, and the solution of problem (3) depends substantially on $\varphi\in[0,\ 2\pi]$.

If we assume that the gas line is an infinite thin rod, then instead of (3) we have a plane problem (if t is not taken into account)

$$\begin{gathered}H(\vartheta)_t=(K\vartheta_x)_x+(K\vartheta_r)_r,\quad 0<x<L,\quad r_0<r<r_1,\quad 0<t\leqslant t_f,\\ \vartheta|_{t=0}=\vartheta_0(x,\ r),\\ K\vartheta_r|_{r_0+\delta_{me}-0}=K\vartheta_r|_{r_0+\delta_{me}+0},\\ K\vartheta_r|_{r_0}=\alpha(\vartheta|_{r_0}-T),\quad K\vartheta_r|_{r_1}=\varphi,\\ \vartheta|_{r_0+\delta_{me}-0}=\vartheta|_{r_0+\delta_{me}+0},\quad K\vartheta_x|_{x=0}=K\vartheta_x|_{x=L}=0,\end{gathered}\tag{9}$$

where $\varphi=0$ for pipelines with $r_0\le 0.2$ m and $\varphi=\alpha_a(T_a-\vartheta)$ if the gas line is considered as a rod; $\tilde{T}=\tilde{T}(x,\ t)$ is determined by (4).

Since in problem (9) the coefficients H and K with $\vartheta=\vartheta^*$ have a discontinuity, we solve (9) by a method suggested in [9-14].

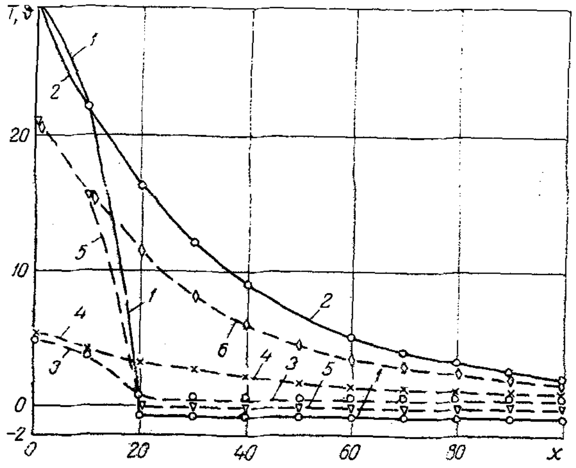

Fig. 1

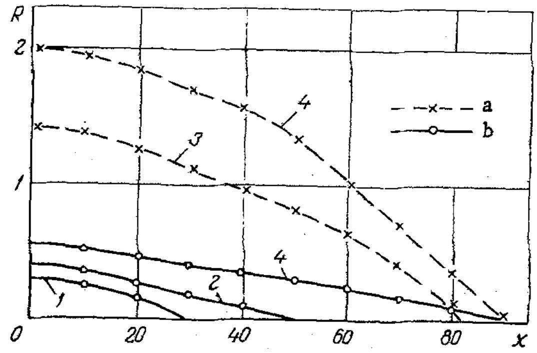

Fig. 2

Fig. 1. Changes in the temperature of the gas T, °C, and of the soil ϑ, °C, along the gas pipeline at different instants of time (1, 2: gas, 3-6: soil): 1) t = 18 h; 2) 198; 3) 18; 4) 198; 5) 18; 6) 198 (3, 4: above the pipe, r = 0.75 m; 5, 6; under the pipe, r = 0.75 m), x, km.

Fig. 2. Change in the depth of thawing of the soil below (a) and above (b) the pipe at different points of the gas pipeline in time: 1) t = 18 h; 2) 56; 3) 250; 4) 720. R, m; x, km.

In accordance with it, the solution of (9) amounts to solving a nonlinear system of algebraic equations by the method of run with iteration [8].

The new method was used in a series of calculations on a digital computer with the following initial data: c_T = 0.6 kcal/kg·deg C; γ_T = 40 kg/m^3; α = 50 kcal/m^2·h·deg C; α_a = 10 kcal/m^2·h·deg C; m = 0.2; $\tilde{\beta}$ = 0.036°C^{-1}; δ_{me} = 0.1 m; $\varkappa$ = 49,920 kcal/m^3;

$$b=\begin{cases} 9\ \text{kcal/m}^3\cdot{}^\circ\text{C} & r_0\leqslant r\leqslant r_0+\delta_{me} \\ 512\ \text{kcal/m}^3\cdot{}^\circ\text{C} & r_0+\delta_{me}\leqslant r\leqslant R, \\ 480\ \text{kcal/m}^3\cdot{}^\circ\text{C} & R<r<r_1, \end{cases}$$

$$K=\begin{cases} 0.003\ \text{kcal/m}\cdot\text{h}\cdot{}^\circ\text{C} & r_0\leqslant r\leqslant r_0+\delta_{me}, \\ 1.88\ \text{kcal/m}\cdot\text{h}\cdot{}^\circ\text{C} & r_0+\delta_{me}\leqslant r<R, \\ 1.16\ \text{kcal/m}\cdot\text{h}\cdot{}^\circ\text{C} & R\leqslant r\leqslant r_1, \end{cases}$$

$$\tilde{\alpha}_a=\alpha_a\left(1+\alpha_a\frac{h_{sn}}{\lambda_{sn}}\right)^{-1};\quad \lambda_{sn}=0{,}3\ \text{kcal/m}\cdot\text{h}\cdot{}^\circ\text{C};$$

$$\vartheta|_{t=0}\equiv-1\,{}^\circ\text{C}=T|_{t=0};\quad r_0=0{,}7\,\text{m};\ r_1=1.25\ \text{m};\quad r_2=25\ \text{m};\quad L=100\ \text{km}$$

Versions of gas transport were examined with cooling to $T_{x=0}$ = 30°C; $T_{x=0} = T_a$ + 5°C; to $T_{x=0}$ = −1°C.

The climatic data (T_a, h_{sn}) used in the calculations are presented in Table 1.

Figure 1 shows the results of calculations of the gas and soil temperatures along the gas pipeline at different instants of time. Curves 1 and 2 correspond to the drop in temperature at the instants of time 18 and 198 h, respectively. The gas temperature before the gas pipeline was put into operation was equal to the soil temperature ($T|_{t=0}$ = −1°C). It can be seen from Fig. 1 that as time passes, the region where gas has above-zero temperature increases. Approximately after 200 h, the gas-temperature distribution along the pipeline is close to steady-state. The time before conditionally steady-state conditions are reached is t ≅ 20 $L\beta_0/w$. With non-steady-state heat exchange, the curve of the gas temperature has a point of discontinuity indicating the position of the front of the heated gas at the corresponding instant of time (curve 1 for t = 18 h). Behind this point, the gas temperature differs from the soil temperature (it increases due to the heat exchange with the warmer soil, $\vartheta|_{t=0}$ = 1°C, but $T|_{t=0}$ = −1°C). Analogous regularities, but with amplitudinal shift, are encountered in the soil under the pipeline (r = 0.75 m, curves 5, 6 in Fig. 1).

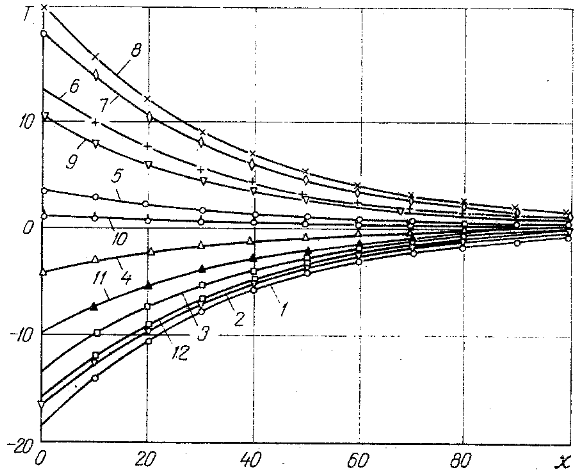

Fig. 3. Change in gas temperature along the gas pipeline in transport of gas cooled by AVO (to $T = T_a + 5°C$) in time: 1-12) January-December, respectively, T, °C.

It can be seen from an analysis of curves 4-6 that in transport of gas with a temperature of 30°C, the soil under the entire pipeline is in a thawed state. Under the pipe the soil temperature, and consequently the depth of thawing, is greater than at the same distance vertically above the pipe. This is due to the influx of cold from the side of the soil surface. In steady state, the gas temperature with increasing distance approaches the soil temperature (within 1-2°C).

Figure 2 shows the change in depth of thawing (below and above the pipe) along a pipeline without heat insulation at different instants of time. With increasing distance, the gas temperature drops, and therefore the depth of thawing decreases. Maximum depth of thawing under the pipe is 2 m; above the pipe it is 0.56 m.

Figure 3 illustrates the change in gas temperature along the gas pipeline with cylindrical heat insulation 0.1 m thick in transport of gas cooled by air coolers (AVO), i.e., for the case when the gas temperature changes synchronously with the air temperature: $T_{x=0} = T_a + \Delta$, where T_a is the arithmetic mean of the gas temperature; Δ is the amplitude of the fluctuations (in the calculations we took $\Delta = 5°C$). It can be seen from Fig. 3 that in the course of a year, the gas temperature on the section from 0 to 50-60 km changes within wide limits (from −18.6 to −3.1°C in January and from 21 to 4.1°C in August).

Over the entire length of the gas pipeline from November to April, the pumped gas has subzero temperature, and from May to October above-zero temperature. With increasing distance, the amplitude of the fluctuations in the gas temperature decreases because of the lower level of the gas temperature and the heat exchange with the air. At the end sections of the gas pipeline ($80 \leq x \leq 100$ km) the gas temperature practically does not change with time.

On the basis of the elaborated model, the thickness of the heat insulation was obtained along the gas pipeline. The optimum thickness of the heat insulation was determined as a result of solving the following problem

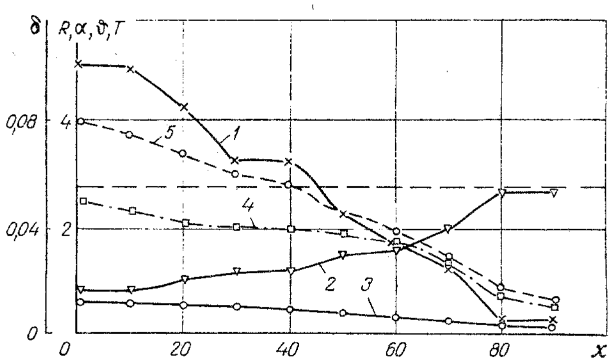

Fig. 4. Change in thickness of the heat insulation, depth of thawing, gas and soil temperature with increasing distance, δ, m; α, kcal/m^2·h·degC; R, m; ϑ/r_0, °C; $T \cdot 10^{-1}$, °C.

[9]: Through the heat insulation, the depth of thawing of the soil $R(x, t)$ had to be limited to the permissible value, proceeding from the condition of ensuring the stability of the pipeline and protection of the environment. It can be seen from Fig. 4 that in this case the maximum depth of thawing is 0.56 m. If we change the thickness of the heat insulation in steps ($\delta_{me} = 0.1$ m for $0 \leq x \leq 30$ km; $\delta_{me} = 0.065$ m for $30 \leq x \leq 40$ km, and $\delta_{me} = 0.05$ for $50 \leq x \leq 90$ km), the depth of thawing of the soil can be limited to 0.56 m at the beginning of the pipeline ($x = 0$) and to 0.2 m at its end ($x = 90$ km). The graph of the gas temperature then has points of inflection at the places of change in $\delta_{me}(x)$.

NOTATION

c_T, A, $c = \gamma_T c_T m + (1-m)c_2\gamma_2$, λ, m, K, α, α_a, $\varkappa$, z_0, R_0, $\bar{\beta}$, ρ, P, w, γ_T are the physical parameters of the gas and the soil;
T, ϑ, T_a are the gas, soil, and air temperature, respectively;
F, r_0 are the cross-sectional area and radius of the pipe, respectively;
L is the length of the pipeline;
r_1 is the distance between the surface of the pipe and the soil surface;
r_2 is the radius of thermal effect;
t_f is the final instant of time;
$\beta_0 = c/\gamma_T c_T$; $\beta = -\beta_1\alpha + \bar{\beta}\beta_2$; $\beta_1 = (2\pi r_0/\gamma_T c_T F)(1/w)$; $\beta_2 = (A\bar{\beta}/\gamma_T c_T)P_x$; $\beta_3 = -(A/\gamma_T/c_T)P_x$; $M = \ln(P/T)$.

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